Conclusions

A simple method based on linear theory has been developed for the determination of trailing- and leading-edge flap deflection schedules to obtain minimum lift-dependent drag (or equivalently maximum lift-to-drag ratio). The resultant lift-dependent drag polar can also be determined. Extensive comparisons with available experimental results have proved the general validity of the method. It is expected that this method would be useful in the preliminary design phase of an aircraft and also in reducing later on the quantum of wind-tunnel testing needed to determine flap schedules.

Acknowledgments

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References

[1] *Subsonic Lift Dependent Drag Due to Boundary Layer of Plane Symmetric Section Wings*, ESDU, Royal Academy of Engineering, 60031-32, 1966.

[2] *RAE Data Sheets, Aerodynamics*, Vol. 2, *Wings*, Royal Academy of Engineering, 01.01.01, 1953 and 01.01.05, 1955.

[3] McKie, J., "The Estimation of the Loading on Swept Wings with Extending Chord Flaps at Subsonic Speeds," Royal Academy of Engineering, TR-69034, Feb. 1969.

[4] Rajeswari, B. and Prabhu, K.R., "Estimation of Flap Schedule and Minimum Drag Envelope," National Aeronautical Laboratory, Bangalore, India, AE-TM-3-83, April 1983, 1977.

[5] Siewert, R. F. and Whitehead, R.E., "Analysis of Advanced Variable Camber Concepts," AGARD-CP-241, 1977.

[6] Webb, T.S., Kent, D.R., and Webb, J.B., "Correlation of F-16 Aerodynamics and Performance Predictions with Early Flight Test Results," AGARD-CP-242.

Bombing Error Sensitivities Using a Simplified Aerodynamic Trajectory

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Nomenclature

C_D	= drag coefficient of bomb
C_I	= crossrange impact point
C_R	= crossrange release point
d	= diameter of bomb
D_I	= downrange impact point
D_R	= downrange release point
E	$= e^{-\alpha T_f}$
h_0	= altitude at bomb release
g	= gravitational constant
K_D	= drag factor
m	= mass of bomb
R_B	= ballistic range
T_f	= time of fall
V_a	= airspeed of bomb at release
V_{ave}	= average velocity during time of flight
V_g	= ground velocity along track
V_{gc}	= ground velocity across track
V_w	= wind velocity
V_z	= component of airspeed in vertical direction
x,y,z	= coordinates
$\dot{x},\dot{y},\dot{z}$	= velocities
$\ddot{x},\ddot{y},\ddot{z}$	= accelerations
α	= factor used to linearize equations of motion
α_0	= average value of α during time of flight
δ	= drift angle
ρ	= air density
ρ_{ave}	= average density during time of flight
θ	= dive angle

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Introduction

ERRORS in determining the initial conditions at bomb release, i.e. airspeed, altitude, vertical velocity, drag coefficient and density, can greatly affect the accuracy at which a bomb reaches the desired impact point. Sensitivity coefficients determined from the solution of the equations of motion define how errors in initial conditions propagate in errors in the downrange and crossrange impact point. Because of the drag term, the equations of motion are nonlinear and thus are not amenable to analytic solutions. However, by using a simplified aerodynamic trajectory (SAT) where the drag term is linearized the sensitivity coefficients can be determined in closed form.

Approach

Referring to Figure 1, the downrange and crossrange impact points respectively can be determined by

$$D_I = D_R + V_g T_f - (V_a T_f \cos\theta - R_B)\cos\delta \tag{1}$$

and

$$C_I = C_R + V_{gc} T_f + (V_a T_f \cos\theta - R_B)\sin\delta \tag{2}$$

By taking the differentials of Eqs. (1) and (2), the total downrange and crossrange error in the impact point can be determined based on the contributions of individual errors in initial conditions. For the downrange impact point

$$\Delta D_I = \Delta D_R + \Delta R_B + (\Delta V_g - \Delta V_a \cos\theta) T_f \tag{3}$$

and for the crossrange

$$\Delta C_I = \Delta C_R + \Delta V_{gc} T_f + (V_a T_f \cos\theta - R_B)\Delta\delta \tag{4}$$

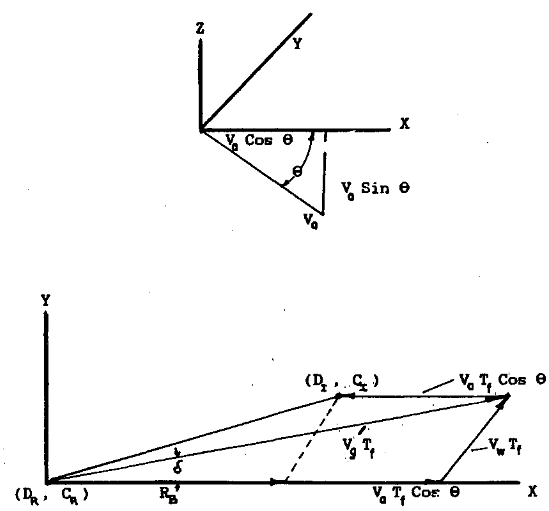

Fig. 1 Coordinate system and geometry for impact prediction.

For the SAT, the drag term in the equations of motion is linearized by assuming that

$$\alpha = \rho_{\text{ave}} K_D V_{\text{ave}} \tag{5}$$

where the values of ρ_{ave}, K_D, and V_{ave} are average values over the extent of the trajectory. The drag factor K_D is defined by

$$K_D = \pi C_D d^2/8m \tag{6}$$

Under this assumption, the equations of motion become

$$\ddot{x} = -\alpha\dot{x}; \qquad \ddot{y} = -\alpha\dot{y}; \qquad \ddot{z} = -\alpha\dot{z} - g \tag{7}$$

Since $R_B = f(V_a \cos\theta, V_z, z, \alpha)$,

$$\Delta R_B = \frac{\partial R_B}{\partial V_a \cos\theta}\Delta V_a \cos\theta + \frac{\partial R_B}{\partial V_z}\Delta V_z + \frac{\partial R_B}{\partial z}\Delta z + \frac{\partial R_B}{\partial \alpha/\alpha_0}\frac{\Delta\alpha}{\alpha_0} \tag{8}$$

After the substitution of Eq. (8), Eqs. (1) and (2) can be written as

$$\Delta D_I = A\Delta D_R + B\Delta V_a \cos\theta + C\Delta V_z + D\Delta z + E\Delta V_g + F\Delta\alpha/\alpha_0 \tag{9}$$

and

$$\Delta C_I = G\Delta C_I + H\Delta V_{gc} + I\Delta\delta \tag{10}$$

where the coefficients are defined in Table 1.

Solving the SAT equations of motion subject to the boundary condition at

$$t=0; \quad x = V_a \cos\theta; \quad y = V_a \cos\theta \sin\delta; \quad z = 0$$

Table 1 Sensitivity coefficients

$A=1$	$B=\partial R_B/\partial V_a \cos\theta - T_f$
$C=\partial R_B/\partial V_z$	$D=\partial R_B/\partial z$
$E=T_f$	$F=\partial R_B/\partial\alpha/\alpha_0$
$G=1$	$H=T_f$
$I=V_a T_f \cos\theta - R_B$	

Table 2 Comparison of results

Conditions	Time of flight, s	Range, ft	$\frac{\partial R_B}{\partial V_a}$, s	$\frac{\partial R_B}{\partial V_z}$, s	$\frac{\partial R_B}{\partial h_0}$	$\frac{\partial R_B}{\partial \alpha/\alpha_0}$, ft
400[a]						
0[b]	13.74[d]	9121	13.30	20.31	1.50	−94
3000[c]	13.74[e]	8978	13.47	19.88	1.47	−121
−45	5.36	2540	5.32	3.91	0.73	−2
3000	5.41	2531	5.37	3.91	0.73	−2.8
800						
0	18.32	22020	14.60	33.11	2.03	−1897
5000	18.12	22221	14.67	34.94	2.10	−1246
−45	5.00	4607	4.81	4.12	0.85	−16
5000	5.00	4604	4.88	4.16	0.85	−11
400						
0	25.19	16555	23.97	19.89	0.81	−275
10000	25.21	16237	24.35	19.31	0.79	−375
−45	14.35	6733	14.06	7.16	0.50	−33
10000	14.45	6680	14.17	7.06	0.50	−44
800						
0	37.49	42866	27.74	29.78	0.94	−5287
20000	37.14	42323	27.99	29.99	0.95	−4537
−45	17.68	15329	15.84	9.89	0.60	−396
20000	17.50	15308	16.24	9.82	0.61	−304

[a]Airspeed in knots. [b]Dive angle in deg. [c]Altitude in ft. [d]Data in this row from True Aerodynamic Trajectory. [e]Data in this row from Simplified Aerodynamic Trajectory.

and at

$$t = T_f; \qquad z = -h_0$$

the range is given by

$$R_B = \left(V_a \cos\frac{\theta}{\alpha}\right)\left[\frac{-h_0 + gT_f/\alpha}{V_a \sin\theta/\alpha + g/\alpha^2}\right] \tag{11}$$

and the time of flight T_f is determined by solving the equation

$$-h_0 + gT_f/\alpha = (V_a \sin\theta/\alpha + g/\alpha^2)(1-E) \tag{12}$$

(Since the expression $e^{-\alpha T_f}$ appears frequently, it will be defined as E.)

The range depends on the airspeed at launch according to

$$\frac{\partial R_B}{\partial V_a \cos\theta} = \frac{(1-E)}{\alpha} \tag{13}$$

Likewise, the vertical launch velocity affects the range by

$$\frac{\partial R_B}{\partial V_z} = \frac{V_a \cos\theta E}{g - V_a \sin\theta[E/(1-E)]} \tag{14}$$

The range depends on the initial altitude according to

$$\frac{\partial R_B}{\partial z} = \frac{V_a \cos\theta E}{g/\alpha - (V_a \sin\theta + g/\alpha)E} \tag{15}$$

Finally, the range is affected by ρ according to

$$\frac{\partial R_B}{\partial \alpha/\alpha_0} = V_a \sin\theta\frac{E-1}{\alpha} + T_f E + E\frac{dT_f}{d\alpha} \tag{16}$$

where

$$\frac{dT_f}{d\alpha} = \frac{-2\alpha h + gT_f - V_a \sin\theta(1-E) - T_f E(\alpha V_a \sin\theta + g)}{\alpha g(E-1) + \alpha^2 V_a \sin\theta E} \tag{17}$$

From Eq. (5), the dependencies on errors in ρ and C_D are

$$\frac{\partial R_B}{\partial C_D/C_{D_0}} = \frac{\partial R_B}{\partial \alpha/\alpha_0} = \frac{\partial R_B}{\partial V_{\text{ave}}/V_{\text{ave}_0}} = \frac{\partial R_B}{\partial \rho/\rho_0}$$

where C_{D_0}, V_{ave_0}, and ρ_{ave_0} are the nominal values used in determining α_0.

Results

Table 2 shows a comparison of results between the SAT discussed here and the solution of the exact equations of motion that include the nonlinear drag term.[1] As might be expected, the results are most similar when bombing from low altitudes but are still quite satisfactory at high altitudes.

Reference

[1]Duke, A. H., "A Ballistic Trajectory Algorithm for Digital Airborne Fire Control," NWC 5416, Naval Weapons Center, China Lake, CA, 1972.